Mathematics 30

Diploma Examination Results
Examiners' Report for January 1996

School-Awarded Mark

A: 25.5	B: 34.9	C: 31.1	F: 8.5

Diploma Examination Mark

A: 18.6	B: 25.9	C: 28.8	F: 26.7

Final Course Mark

A: 20.7	B: 31.2	C: 34.9	F: 13.2
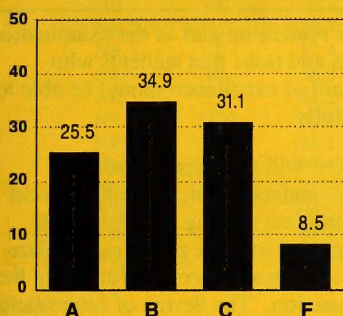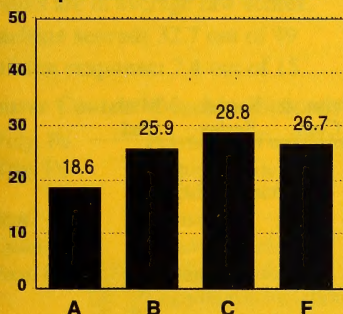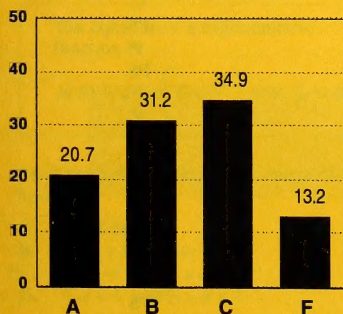
The summary information in this report provides teachers, school administrators, students, and the general public with an overview of results from the January 1996 administration of the Mathematics 30 Diploma Examination. This information is most helpful when used with the detailed school and jurisdiction reports that have been mailed to schools and school jurisdiction offices. An annual provincial report containing a detailed analysis of the combined January, June, and August results is published each year.

Description of the Examination

The Mathematics 30 Diploma Examination consists of three parts: a multiple-choice section of 40 questions and a numerical-response section of nine questions, together worth 70%, and a written-response section of three questions, worth 30% of the total examination mark.

Achievement of Standards

The information reported is based on the final course marks achieved by 9 614 students who wrote the January 1996 examination.

- 86.8% of the 9 614 students achieved the acceptable standard (a final course mark of 50% or higher).
- 20.7% of these students achieved the standard of excellence (a final course mark of 80% or higher).

Approximately 50% of the students who wrote the January 1996 examination were females.

- 86.6% of the female population achieved the acceptable standard (a final course mark of 50% or higher).
- 18.8% of these students achieved the standard of excellence (a final course mark of 80% or higher).

Approximately 50% of the students who wrote the January 1996 examination were males.

- 87.0% of the male population achieved the acceptable standard (a final course mark of 50% or higher).
- 22.6% of these students achieved the standard of excellence (a final course mark of 80% or higher).

Students First!
Student Evaluation

Alberta
EDUCATION

Provincial Averages

- The average school-awarded mark was 68.4%.
- The average diploma examination mark was 61.6%.
- The average final course mark, representing an equal weighting of the school-awarded mark and the diploma examination mark, was 65.4%.

Of the 9 614 students who wrote the January 1996 examination, 1 575 had written at least one Math 30 exam previously.

Results and Examiners' Comments

This examination has a balance of question types and difficulties reflecting the philosophy of the Mathematics 30 Course of Studies. It was designed so that students who are achieving the acceptable standard in Mathematics 30 should obtain a mark of 50% or higher. Students who are achieving the standard of excellence in Mathematics 30 should obtain a mark of 80% or higher. Students who are achieving the acceptable standard or the standard of excellence are expected to be able to achieve the curriculum standards identified in the *Mathematics 30 Information Bulletin, Diploma Examination Program*. At least 70% of the examination includes questions and tasks that students who achieve the acceptable standard should be able to complete

successfully. The remaining part of the examination includes questions and tasks that students who achieve the standard of excellence should be able to complete successfully.

Future examinations will continue to focus on assessing students' understanding of mathematical concepts and on problem solving. Students will continue to be expected to solve problems, explain solutions, justify solutions, or prove solutions in the written-response section. The design of future field tests and examinations will include items that assess how well students have achieved the general learner expectations stated in the Mathematics 30 Course of Studies.

Blueprint

Question	Key	Difficulty	Poly. Fn.	Trig. Fn.	Stat.	Quad. Rltns.	Exp. & Log.	Perm. & Com.	Seq. & Series	Math Und.
MC 1	B	80.7	√							P
MC 2	D	80.3	√							C
MC 3	A	74.0	√							PS
MC 4	C	92.8	√							C
MC 5	D	54.4	√							PS
MC 6	D	63.2	√							C
MC 7	A	78.9	√							C
MC 8	D	88.0		√						P
MC 9	B	79.6		√						C
MC 10	C	59.4		√						PS
MC 11	C	67.4		√						P
MC 12	D	63.9		√						C
MC 13	A	62.8		√						PS
MC 14	C	56.3		√						P
MC 15	B	77.8					√			PS
MC 16	A	84.8					√			P
MC 17	B	70.5					√			P
MC 18	C	66.6					√			P
MC 19	B	78.2					√			P
MC 20	A	37.9					√			PS
MC 21	D	85.7					√			P
MC 22	A	72.9				√				P
MC 23	D	73.7				√				C
MC 24	C	52.0				√				PS
MC 25	B	58.9				√				PS
MC 26	A	75.1				√				C
MC 27	D	86.0				√				C
MC 28	A	64.5							√	PS
MC 29	A	83.8							√	C
MC 30	D	74.1							√	P
MC 31	C	41.8							√	PS
MC 32	B	56.7							√	C

Question	Key	Difficulty	Poly. Fn.	Trig. Fn.	Stat.	Quad. Rltns.	Exp. & Log.	Perm. & Com.	Seq. & Series	Math Und.
MC 33	B	28.5							√	PS
MC 34	D	69.7					√			P
MC 35	A	62.5					√			C
MC 36	C	59.3					√			PS
MC 37	D	36.8					√			PS
MC 38	A	77.1		√						PS
MC 39	C	66.8		√						C
MC 40	A	84.9		√						C
NR 1	360	76.7						√		C
NR 2	24.0	32.7							√	P
NR 3	15	57.4						√		C
NR 4	20	56.8	√							C
NR 5	5.5	81.2				√				P
NR 6	2200	60.9						√		P
NR 7	0.25	59.1				√				PS
NR 8	1.43	71.4		√						PS
NR 9	31.4	41.5			√					PS
WR 1	—	52.6								PCPS
WR 2	—	41.8								PCPS
WR 3	—	54.0								PCPS

Subtest

When analyzing detailed results, please bear in mind that subtest results **cannot** be directly compared.

Results are in average raw scores.

Machine scored: 32.7 out of 49

Written response: 7.4 out of 15

Course Content (Machine Scored)

Poly. Fn.	Polynomial Functions	5.8	out of 8
Trig. Fn.	Trigonometric and Circular Functions	5.2	out of 8
Stat.	Statistics	3.0	out of 4
Quad. Rltns.	Quadratic Relations	4.8	out of 7
Exp. & Log.	Exponential and Logarithmic Functions	5.8	out of 8

Perm. & Com.	Permutations and Combinations	3.8	out of 7
Seq. & Series	Sequences and Series	4.3	out of 7

Mathematical Understandings*

- Procedural (P): 10.7 out of 15
- Conceptual (C): 12.4 out of 17
- Problem Solving (PS): 9.6 out of 17

*Refer to Appendix D of the 1995–96 *Mathematics 30 Information Bulletin, Diploma Examinations Program*, for an explanation of mathematical understandings. These are the mathematical abilities described in Appendix G.

The equation of a trigonometric function is

$$h(\theta) = a \sin\left(\theta + \frac{\pi}{2}\right) \pm 4, \quad a > 0$$

D. An expression that defines the range of this function is

- A. $-a \le h(\theta) \le a$
- B. $-4a \le h(\theta) \le 4a$
- •C. $-a - 4 \le h(\theta) \le a - 4$
- D. $-a + 4 \le h(\theta) \le a + 4$

Multiple-Choice and Numerical-Response Questions

The multiple-choice and numerical-response sections of the examination ask a sample of questions that cover all content areas in Mathematics 30. A discussion about how well students achieve the curriculum standards in the units of Trigonometric and Circular Functions and Sequences and Series follows.

Trigonometric and Circular Functions — In order to achieve the curriculum standards for this unit, students should be able to convert the angle measurements between degree and radian measure, verify the fundamental trigonometric identities, solve first-degree trigonometric equations on the domain $0 \le \theta < 2\pi$ in radians and $0° \le \theta < 360°$, and simplify and evaluate simple trigonometric expressions involving the fundamental trigonometric identities. Students should also be able to generate the graph of trigonometric functions with the use of graphing calculators or graphing utility packages and explain the effect of each parameter a, b, c, and d on the graphs of the functions $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$. In addition, students should be able to state the domain and

A circle with centre C is shown below.
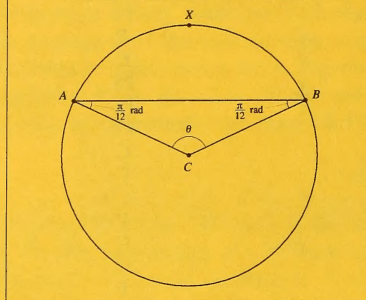
9. If the radius of the circle is 12 cm, then correct to the nearest tenth of a centimetre, the length of arc AXB is _____ .
(Record your answer on the answer sheet.)

Answer: **31.4**

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28. The general term of a sequence is $t_n = 2(3^n)$, $n \in N$. A recursive definition for this sequence is

•A. $\begin{cases} t_1 = 6 \\ t_n = 3\Sigma t_{n-1}, \ n > 1 \end{cases}$

B. $\begin{cases} t_1 = 6 \\ t_n = 6\Sigma t_{n-1}, \ n > 1 \end{cases}$

C. $\begin{cases} t_1 = 3 \\ t_n = 3\Sigma t_{n-1}, \ n > 1 \end{cases}$

D. $\begin{cases} t_1 = 3 \\ t_n = 6\Sigma t_{n-1}, \ n > 1 \end{cases}$

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33. The graphical representation of the integer sum of the terms of an arithmetic sequence t_n, $n \in N$, is shown below. Each point on the graph is in the form (n, S_n), where $n \in N$ and S_n is the sum of the first n terms. For example, the point $(4, -4)$ represents $S_4 = -4$.
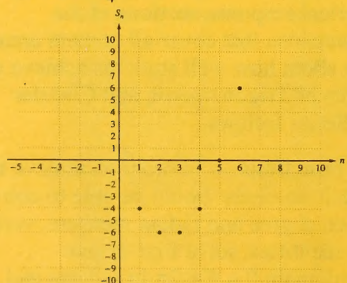
In this sequence, the term t_n, which equals zero, is

A. t_2
•B. t_3
C. t_5
D. t_6

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range of $y = \sin\theta$, $y = \cos\theta$, and $y = \tan\theta$ and describe, orally and in writing, the relationship between the root(s) of a first-degree trigonometric equation and the graph of its corresponding function, as well as participate in and contribute toward the problem-solving process for problems that can be represented by trigonometric functions studied in Mathematics 30. Multiple-choice questions 8 to 14 and numerical-response question 9 required students to demonstrate their understanding of this unit. Of the students who achieved the acceptable standard but not the standard of excellence, 89.4% answered multiple-choice question 8 correctly; 79.7% answered question 9 correctly; 55.5% answered question 10 correctly; 64.9% answered question 11 correctly; 63.0% answered question 12 correctly; 59.9% answered question 13 correctly; and 52.5% answered question 14 correctly. However, only 36% of these students achieved the expectations of numerical-response question 9.

Students who achieve the standard of excellence are expected to be able to prove trigonometric identities and to explain, orally and in writing, the combined effects of the parameters $a, b, c,$ and d in the trigonometric functions $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$, on the functions' domain and range. They are also expected to solve first- and second-degree trigonometric equations including double and half angles on the domain $0 < \theta \leq 2\pi$ or $0° \leq \theta < 360°$, as well as describe, orally and in writing, the relationship between the root(s) of a trigonometric equation and the graph of its corresponding function, and complete solutions to problems that can be represented by trigonometric functions studied in Mathematics 30. Multiple-choice questions 10, 11, and 14 required students to show that they can do this. Of the students who achieved the standard of excellence on the examination, 87.0% answered multiple-choice question 10 correctly, 90.5% answered multiple-choice question 11 correctly, and 83.2% answered multiple-choice question 14 correctly. Multiple-choice questions 8, 9, 12, and 13 and numerical-response question 9 identified expectations for those students who achieved the acceptable standard but not the standard of excellence; over 80% of students who achieved the standard of excellence on the examination were able to successfully achieve the expectations of these questions.

Sequences and Series — To achieve the acceptable standard in sequences and series, students must be able to write the specific terms of a sequence, given its defining function; expand a series given in sigma notation; describe, orally and in writing, the difference between sequences and series, arithmetic and geometric, infinite and finite; and, apply the general term "formula" to arithmetic and geometric sequences. They must also be able to apply the sum formula for arithmetic and geometric series and be able to participate in and contribute toward the problem-solving process for problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30. Multiple-choice questions 28 to 33 and numerical-response question 2 required students to demonstrate their understanding of this unit. Of the students who achieved the acceptable standard but not the standard of excellence, 61.7% correctly answered multiple-choice question 28; 83.6% correctly answered question 29; 74.2% correctly answered question

; 38.2% correctly answered question 31; 53.0% correctly answered question 32; and 20.7% correctly answered question 33. On numerical-response question 25.1% of these students answered correctly.
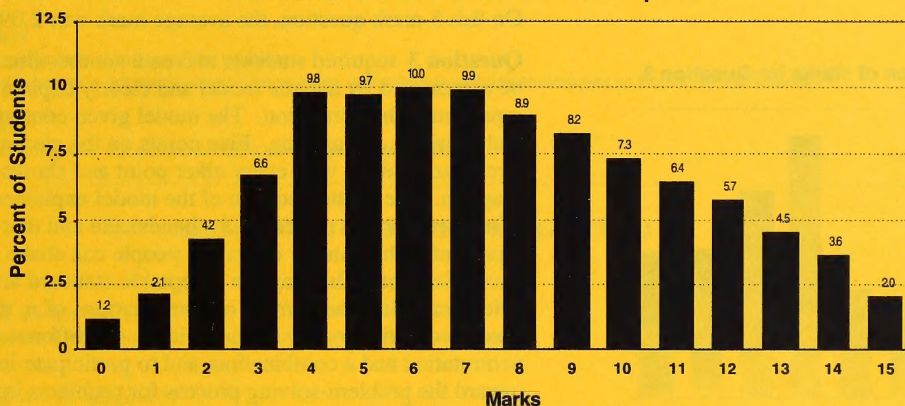
In addition to the expectations for the acceptable standard, students who achieve the standard of excellence must be able to solve problems using the general term and/or sum formulas in which there are two unknowns, write the specific terms of a sequence

when given its recursive definition, determine the functions describing any sequence that has a recognizable pattern, and complete the solution to problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30. Multiple-choice questions 28, 30, and 33 require this of students. Of the students who achieved the standard of excellence, 94.6% answered multiple-choice 28 correctly; 97.7 answered question 30 correctly; and 66.2% answered question 33 correctly.
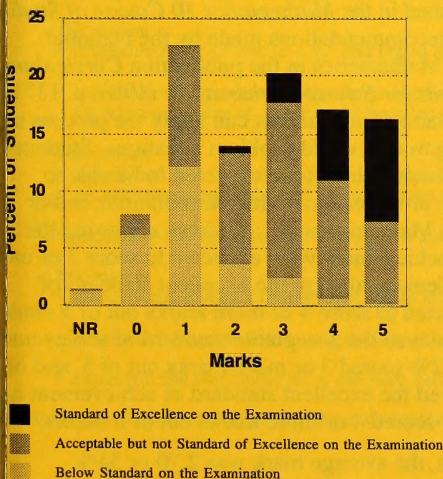
Written-Response Question

As published in the 1994–95 and 1995–96 *Mathematics 30 Diploma Examination Information Bulletins*, the written-response questions assess whether or not students can draw on their mathematical experiences to solve problems and to explain mathematical concepts. Therefore, the written-response questions do not necessarily fall into a particular unit of study but may cross more than one unit or may require students to make connection between mathematical concepts. Students achieving at the acceptable standard were expected to obtain at least half marks on all questions. Students achieving at the standard of excellence were expected to answer all questions almost perfectly.

Distribution of Marks for Written Response

Distribution of Marks for Question 1

■ Standard of Excellence on the Examination

▦ Acceptable but not Standard of Excellence on the Examination

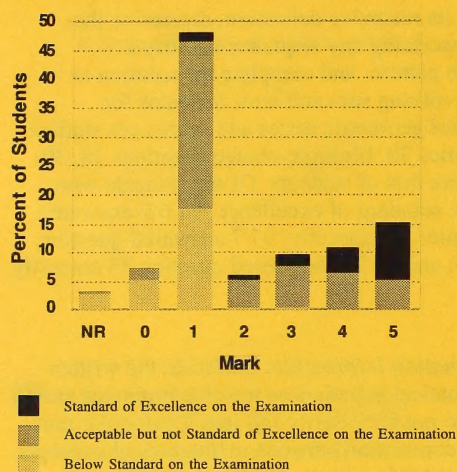▨ Below Standard on the Examination

Question 1 required students to find two different values of n that satisfy $S_n = 612$ if the first term of an arithmetic sequence is 84 and the common difference is –6. Students were also asked to explain why two different values for the number of terms can yield the same sum. Students who achieve the acceptable standard in Mathematics 30 are expected to participate in the problem-solving process for problems involving the sum and term formulas for arithmetic and geometric series and sequences. Students who achieve the standard of excellence are expected to complete the solution to such problems. Students who achieved the acceptable standard were expected to determine the two values for n and link the solution to summation. It was expected that students achieving this standard of achievement would score 3 out of 5 marks. Of the students who met the acceptable standard of achievement on the examination, 51.3% received at least 3 out of 5 marks. Students who achieved the excellent standard of achievement on the examination were expected to score 4 or 5 marks out of 5 marks.

On this 5-mark question, the average mark was 2.63 or 52.6%.

Distribution of Marks for Question 2

Chart: Percent of Students (y-axis, 0 to 50) vs Mark (x-axis: NR, 0, 1, 2, 3, 4, 5)

Legend:
- Standard of Excellence on the Examination
- Acceptable but not Standard of Excellence on the Examination
- Below Standard on the Examination

Distribution of Marks for Question 3

Chart: Percent of Students (y-axis, 0 to 30) vs Marks (x-axis: NR, 0, 1, 2, 3, 4, 5)

Legend:
- Standard of Excellence on the Examination
- Acceptable but not Standard of Excellence on the Examination
- Below Standard on the Examination

Question 2 required students to algebraically show why the only solutions to the equation $\log_x(19x - 30) = 3$ are $x = 2$ and $x = 3$. The written-response question crossed the units of Exponential and Logarithmic Functions and Polynomial Functions. Students demonstrating acceptable achievement are expected to be able to convert from logarithmic form to exponential form and to solve and verify simple exponential and logarithmic equations, as well as state the domain and range of the exponential and logarithmic functions. These students can also factor and find the zeros of an integral polynomial function of degree 3 or less, in which all the zeros are rational. Students demonstrating excellent achievement are expected to be able to complete solutions to problems that involve polynomial functions and exponential and logarithmic functions studied in Mathematics 30. It was expected that students achieving the acceptable standard of achievement would score 3 or more marks; whereas, students demonstrating excellent achievement would score 4 or more marks out of 5 marks on this question. Of the students demonstrating the acceptable standard of achievement on the examination, 27.9% scored 3 or more marks out of 5 marks; whereas of the students demonstrating excellent achievement, 71.7% scored or more marks out of 5 marks.

On this 5-mark question, the average mark was 2.09 or 41.8%.
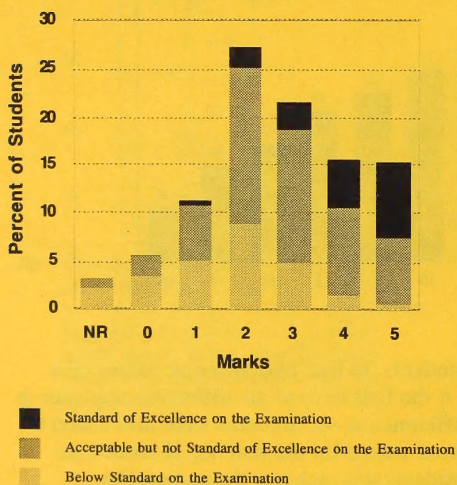
Question 3 required students to create another situation that could be represented by a given model and clearly explain how the model represented their situation. The model given consisted of a diagram and a written explanation. Five points on the circumference of a circle were joined with every other point and shown in a given diagram. The written portion of the model explained that the line joining two points represented a handshake and that the model represented the number of ways 5 people can shake each other's hand once. Students achieving the acceptable standard are expected to be able to calculate the number of combinations of n things taken r at a time; to describe, orally and in writing, the difference between a permutation and a combination; and to participate in and contribute toward the problem-solving process for problems involving permutations and/or combinations studied in Mathematics 30. As detailed in the 1995–96 *Mathematics 30 Information Bulletin* (p.16) the diploma examination reflects mathematics as problem solving. The expectations contained in the *Mathematics 30 Course of Studies* are consistent with the recommendations made by the National Council of Teachers of Mathematics in the publication *Curriculum and Evaluation Standards for School Mathematics* (1989, p. 137). These include an expectation that students can apply the process of mathematical modelling to real-world problem situations. Students demonstrating excellent achievement are expected to be able to complete the solution to problems involving permutations and/or combinations studied in Mathematics 30. Students achieving the acceptable standard of achievement were expected to score 3 or more marks out of 5, and students achieving the excellent standard of achievement were expected to score 4 or more marks out of 5 marks. Of the students who achieved the acceptable standard of achievement on the examination, 49.2% scored 3 or more marks out of 5, and of the students who achieved the excellent standard of achievement on the examination, 64.1% scored 4 or more marks out of 5 marks.

On this 5-mark question, the average mark was 2.70 or 54%.

redit may be given to students who show unusual insight. If their solutions fall outside Specific Question Scoring *ubrics*, they will be scored against the *General Scoring Guide* shown below.

GENERAL SCORING GUIDE

marks The student
  - demonstrated a *complete understanding* of the problem
  - used mathematical knowledge and problem-solving techniques to find the solution
  - justified the solution and explained its relevance to the problem

marks The student
  - demonstrated *an understanding* of the problem
  - chose a strategy that used mathematical knowledge and problem-solving techniques to find a solution, but the procedure contained *a minor flaw*
  - showed *some justification* of his or her results

marks The student
  - demonstrated *some understanding* of the problem
  - formulated *some aspects* of the problem mathematically
  - demonstrated the use of a strategy that used mathematical knowledge and problem-solving techniques to find a *partial* solution
  - communicated little understanding of the complexities of the problem

marks The student
  - explored the *initial stages* of the problem
  - applied *some* relevant mathematical knowledge and problem-solving techniques to find a *partial* solution

mark The student
  - applied some relevant mathematical knowledge to the problem

SPECIFIC QUESTION SCORING RUBRICS

uestion 1

The student
  - determined $n = 12$ and $n = 17$
  - clearly explained why two different values for n can yield the same sum*

The student
  - determined $n = 12$ and $n = 17$
  - did not clearly explain why two different values for n can yield the same sum

  or
  - attempted to find $n = 12$ or $n = 17$, but the strategy contained a minor error
  - clearly explained why two different values for n can yield the same sum*

The student
  - determined $n = 12$ and $n = 17$ and linked the solution to **summation**

  or
  - attempted to determine value(s) for n but did not complete the procedure, although a strategy was evident. Provided an explanation for why two values of n can yield the same sum.*

The student
  - determined $n = 12$ and $n = 17$ with supporting work
  - did not attempt to explain why two different values for n can yield the same **sum**

  or
  - explained why two different values for n can yield the same sum, but the explanation lacked clarity*

The student
  - selected and applied a strategy that would have led to the correct values for n if it had been carried out; e.g., wrote out the terms

  or
  - substituted values from the question into an arithmetic sum formula

  or
  - explained that two values of n can exist because the difference is negative

Student does not have to use the arithmetic sequence given in the question to explain why two different value for n can yield the same sum.

Question 2

5   The student
- demonstrated a correct algebraic procedure or explanation to solve $\log_x (19x - 30) = 3$
- explained why $x = -5$ is not part of the solution

4   The student
- demonstrated a correct algebraic procedure or explanation to solve $\log_x (19x - 30) = 3$

  and
- gave a weak explanation of why $x = -5$ is not part of the solution

3   The student
- demonstrated a correct algebraic procedure or explanation to find the solutions to $0 = x^3 - 19x + 30$ to be 2, 3, and -5

  or
- found the solution to $\log_x (19x - 30) = 3$ with no procedure shown and explained why $x = -5$ is not part of the solution

2   The student
- found the solutions of $\log_x (19x - 30) = 3$ to be 2, 3, and -5, with no procedure shown

  or
- demonstrated a correct algebraic procedure or explanation to find the solutions to $0 = x^3 - 19x + 30$, but made an error

1   The student
- started a procedure, e.g., changed the equation to exponential form

  or
- verified by substitution that $x = 2$ and $x = 3$ satisfy the equation $\log_x (19x - 30) = 3$. This is not sufficient to show why the solution consists of only 2 values for x.

Question 3

5   The student
- created another situation that is represented by **this** model
- clearly explained **how** the model represents it
- included a clear discussion of "order" and "total number of lines"

4   The student
- created another situation that is represented by **this** model, but
- did not **clearly** communicate **how** the model represents the situation

  or
- clearly communicated **how** his or her model represents a situation that was not $_5C_2$

3   The student
- only created another situation **that is** represented by **this** model, and
- did not explain **how** this model works

  or
- provided another situation that **could be** represented by this *diagram* **and** clearly explained how it worked

2   The student
- demonstrated **some** understanding of what the model represented (for example, $_5C_2$), but did not provide another situation that could represent the model

  or
- provided another situation that **could be** represented by this *diagram* but not the presented model

1   The student
- provided a situation that could have some connection to the diagram

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DATE DUE SLIP

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